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New Formula Relating the Yield Stress-Strain With the Strength Coefficient and the Strain-Hardening Exponent

Zhang Zhongping, Wu Weihua, Chen Donglin, Sun Qiang, and Zhao Wenzhen

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The main purpose of this paper is to search formulas for different metals that relate the yield stress-strain with the strength coefficient and the strain-hardening exponent. For this purpose, the test data of nine alloys were used as basic data and the applicability of Hollomon's equation at the yield point of the alloy was studied. This paper explores new equations relating the yield stress-strain with the strength coefficient and the strain-hardening exponent. At the same time, the study introduces a new fracture-ductility parameter. The new fracture-ductility parameter not only describes the applicability of the new equations to these alloys, but may also be better at describing the hardening behavior of a metallic material.

Keywords fracture ductility parameter, Hollomon's equation, strain-hardening exponent, strength coefficient, yield stress-strain

1. Introduction

Whether one is predicting metal fatigue crack initiation life through the equivalent stress-amplitude method[1-3] or studying metal tensile properties, two basic material constants must be known: the strength coefficient and the strain-hardening exponent. Although these two constants can be determined experimentally, they are frequently calculated theoretically using yield stress-strain data, because comprehensive tensile testing is expensive and time consuming. The traditional formula used to correlate yield stress-strain data with the strength coefficient and the strain-hardening exponent has been Hollomon's equation.[4] However, the following discussion shows that Hollomon's equation does not accurately express the relationship among yield stress-strain, the strength coefficient, and the strain-hardening exponent. What is more, that formula is not applicable to some kinds of alloys. Therefore, to theoretically obtain the strength coefficient and the strain-hardening exponent as accurately as possible, it is first necessary to know the applicable conditions of the Hollomon equation at the yield point of an alloy. Then, an effort should be made to develop appropriate relations correlating the yield stress-strain data with the strength coefficient and the strain-hardening exponent.

Thus, in this paper, test results on nine alloys[5-7] are taken as basic mechanical property data. The applicability of the Hollomon equation at the yield point of the alloy was then studied, and a new relation was developed correlating the yield stress-strain with the strength coefficient and the strain-hardening exponent. In addition, a new fracture-ductility parameter is introduced.

Zhang Zhongping and **Zhao Wenzhen,** The State Key Laboratory for Mechanical Behavior of Materials, Xi'an Jiaotong University, Xi'an, China, 710049; **Zhang Zhongping, Wu Weihua, Chen Donglin,** and **Sun Qiang,** Air Force Engineering University, Xi'an, China, 710051. Contact e-mail: zhangzhp1962@eyou.com.

2. Stress-Strain Relation at the Yield Point

It is well known that the Hollomon equation[8] is given by the following equations:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{k}\right)^{1/n} \quad \text{(Eq 1)}$$

and

$$\sigma = k\varepsilon_p^n \quad \text{(Eq 2)}$$

In Eq 1 and 2, σ is the stress, ε is the total strain, ε_p is the plastic strain, E is Young's modulus, k is the strength coefficient, and n is the strain-hardening exponent.

If Eq 2 is used to evaluate the yield stress-strain behavior of an alloy, and if the plastic yield strain (ε_p) is equal to 0.002, then there is

$$\sigma_{0.2} = k(0.002)^n \quad \text{(Eq 3)}$$

where $\sigma_{0.2}$ is the yield stress.

Nomenclature

E	Young's modulus
k	strength coefficient
n	strain-hardening exponent
ε	total strain
ε_f	the fracture strain
ε_p	plastic strain
σ	stress
σ_b	the ultimate tensile stress
σ_f	the fracture stress
$\sigma_{0.2}$	true yield stress
$\sigma'_{0.2}$	theoretical yield stress
α	new fracture ductility parameter
ψ	the reduction of area

Equation 3 has been used in Ref 4 to calculate the strength coefficient and the strain-hardening exponent. The results have been used as basic data in predicting metal fatigue crack initiation life by the equivalent stress-amplitude method.[1-3]

Equations 1 and 2 are fit to the tensile data (σ, ε), and deviation problems arise when they are used at specific points (e.g., $\sigma_{0.2,}$ 0.002). That is to say, Eq 3 may not properly express the relationship between the yield stress and the plastic yield strain. The correctness of Eq 3 must be checked for universal applicability. In Ref 5-7, performance parameters from experiments on nine alloys are given. The performance parameters include factors such as yield stress, the ultimate tensile stress, the strength coefficient, and the strain-hardening exponent. If both the strength coefficient and the strain-hardening exponent given in Ref 5-7 are taken as true values, then these values can be substituted into Eq 3. The result from this exercise can be considered as the "theoretical yield stress." The yield stress given in Ref 5-7 is then referred to as the "true yield stress." The applicability of Eq 3 can be determined by comparing the "theoretical yield stress" with the "true one."

The performance parameters and the theoretical yield stress calculated from Eq 3 are listed in Table 1 and 2. From the data in these tables, ψ is the reduction of area, σ_f is the fracture stress, ε_f is the fracture strain, σ_b is the ultimate tensile stress, $\sigma_{0.2}$ is the true yield stress, $\sigma'_{0.2}$ is the theoretical yield stress, and $\delta\sigma'_{0.2} = (\sigma'_{0.2} - \sigma_{0.2})/\sigma_{0.2}$. The units of σ_f, $\sigma_{0.2,}$ $\sigma'_{0.2}$, $\sigma_{b,}$ and k are MPa.

It can be seen that for the alloys listed in Table 1, the theoretical yield stress as determined from Eq 3 is approximately equal to the values determined from the mechanical test program. However, for certain alloys in Table 2, the theoretical yield stress deviates greatly from the experimentally determined ones. So, for certain of the alloys (Table 1), the Hollomon equation approximately expresses the relationship among yield stress-strain, strength coefficient, and strain-hardening exponent. For those alloys shown in Table 2, the Hollomon equation does not adequately express the relationship among these parameters.

If the data in these two tables are examined closely, it is found that for the alloys in Table 1:

$$k < \sigma_f \qquad \text{(Eq 4)}$$

$$5\% < \psi\, \varepsilon_f < 10\% \qquad \text{(Eq 5)}$$

or

Table 1 Parameters of the Alloys[5] and the Theoretical Yield Stresses

Material	Ly12cz(plate)	Lc9cgs3	30CrMnSiA	30CrMnSiNi2A	$40CrMnSiMoV_A$
ψ, %	26.6	21.0	53.6	52.3	43.7
ε_f, %	30.2	28.3	77.3	74.0	63.3
σ_f	61.8	748	1795	2600	3512
k	545	724	1476	2355	3150
n	0.089	0.071	0.063	0.091	0.147
σ_b	475.6	560.2	1177.0	1655.4	1875.3
$\sigma_{0.2}$	331.5	518.2	1104.5	1308.3	1513.2
$\psi\varepsilon_f$, %	8.03	5.95	41.41	38.70	27.67
$\sigma'_{0.2}$	313.8	466.1	997.7	1338.0	1265.1
$\delta\sigma'_{0.2}$, %	−5.3	−10.1	−9.7	2.3	−16.4
$\sigma''_{0.2}$	360.4	495.6	1054.2	1436.4	1442.5
$\delta\sigma''_{0.2}$, %	8.7	−4.4	−4.6	9.8	−4.7
$\sigma_b/\sigma_{0.2}$	1.43	1.08	1.07	1.27	1.24
H/S	H	H	S	S	S

H, hardening; S, softening

Table 2 Parameters of the Alloys[5-7] and the Theoretical Yield Stresses

Material	Ly12cz(rod)	Lc4cs	2024-T4	7075-T6
ψ, %	16.5	16.6	35.0	33.0
ε_f, %	18.0	18.0	43.0	41.0
σ_f	643	711	634	745
k	850	775	807	827
n	0.158	0.063	0.2	0.113
σ_b	545.1	613.9	476	579
$\sigma_{0.2}$	399.5	570.8	303	469
$\psi\varepsilon_f$, %	2.97	2.99	15.05	13.53
$\sigma'_{0.2}$	318.3	524.0	233	410
$\delta\sigma'_{0.2}$, %	−20.3	−8.2	−23.1	−12.6
$\sigma''_{0.2}$	394.7	558.2	310	471
$\delta\sigma''_{0.2}$, %	−1.2	−2.2	2.3	0.4
$\sigma_b/\sigma_{0.2}$	1.36	1.08	1.57	1.23
H/S	H	H	H	H

H, hardening; S, softening

$$\psi\ \varepsilon_f > 20\% \quad \text{(Eq 6)}$$

While for the data in the Table 2:

$$k > \sigma_f \quad \text{(Eq 7)}$$

$$10\% < \psi\ \varepsilon_f < 20\% \quad \text{(Eq 8)}$$

or

$$\psi\ \varepsilon_f < 5\% \quad \text{(Eq 9)}$$

That is to say, when inequalities in Eq 4-6 hold, the Hollomon equation approximately expresses the relationship among yield stress-strain, strength coefficient, and strain-hardening exponent. When inequalities in Eq 7-9 hold, the Hollomon equation does not express the relationship among them very well.

Moreover, from the data in Table 2, an empirical relationship can be determined

$$\sigma_{0.2}^{5/3} = \sigma_b^{2/3}\, k(0.002)^n \quad \text{(Eq 10)}$$

To show the correctness and the accuracy of Eq 10, the strength coefficient, the strain-hardening exponent, and the ultimate tensile stress given in Ref 5-7 are taken as true values and substituted into Eq 10. The calculated result is called as the "theoretical yield stress" and is denoted as $\sigma''_{0.2}$. The value given in Ref 5-7 is called as the "true yield stress" for this discussion. By comparing the theoretical yield stress with the true one, and defining $\delta\sigma''_{0.2} = (\sigma''_{0.2} - \sigma_{0.2})/\sigma_{0.2}$, it can be seen that the maximum relative error is only 2.3%. Therefore, Eq 10 numerically expresses the relationship among yield stress-strain, strength coefficient, and strain-hardening exponent when $k > \sigma_f$, or $\psi\ \varepsilon_f < 5\%$, or $10\% < \psi\ \varepsilon_f < 20\%$.

As mentioned previously, the Hollomon equation only expresses approximately the relationship among yield stress-strain, strength coefficient, and strain-hardening exponent for the alloys listed in Table 1. Furthermore, the purpose in studying the relationship is to calculate as accurately as possible the strength coefficient and the strain-hardening exponent. Therefore, a more precise expression was sought. Now if the data in Table 1 are further evaluated, then

$$\sigma_{0.2}^{3/2} = \sigma_b^{1/2}\, k(0.002)^n \quad \text{(Eq 11)}$$

Again the $\sigma_{0.2}$ term in Eq 11 is taken as the "theoretical yield stress," which needs to be determined. Thus, the result calculated from Eq 11 is listed in Table 1 as $\sigma''_{0.2}$. When $\sigma''_{0.2}$ is compared with the true yield stress given in Ref 5-7, it can be seen that, relative to Eq 3, Eq 11 more precisely expresses the relationship among yield stress-strain, strength coefficient, and strain-hardening exponent when $k < \sigma_f$, or $5\% < \psi\ \varepsilon_f < 10\%$, or $\psi\ \varepsilon_f > 20\%$.

When Eq 10 and 11 are compared with Eq 3, it was found that $(\sigma_b/\sigma_{0.2})^{2/3}$ appears in Eq 10, and $(\sigma_b/\sigma_{0.2})^{1/2}$ appears in Eq 11. The appearance of $(\sigma_b/\sigma_{0.2})^{1/2}$ in Eq 11 reduces the maximum relative error between the theoretical yield stress and the true one in Table 1 from 16.4% to 9.8%. Similarly, $(\sigma_b/\sigma_{0.2})^{2/3}$ in Eq 10 reduces the maximum relative error in Table 2 from 23.1% to 2.3%. Therefore, Eq 10 and 11 can be considered as the new relations that better correlate yield stress-strain with the strength coefficient and the strain-hardening exponent.

3. Fracture Ductility of Metals and Alloys

If

$$\alpha = \psi\ \varepsilon_f \quad \text{(Eq 12)}$$

then the inequalities of Eq 5 and 6 change to:

$$5\% < \alpha < 10\% \quad \text{(Eq 13)}$$

or

$$\alpha > 20\% \quad \text{(Eq 14)}$$

respectively, for the alloys listed in Table 1, while the inequalities of Eq 8 and 9 change to:

$$10\% < \alpha < 20\% \quad \text{(Eq 15)}$$

or

$$\alpha < 5\% \quad \text{(Eq 16)}$$

respectively, for the alloys listed in Table 2. Because $\varepsilon_f = -\ln(1 - \psi)$, Eq 12 can be changed to:

$$\alpha = -\psi \ln(1 - \psi) \quad \text{(Eq 17)}$$

where ψ is the reduction of area and reflects the fracture ductility of the alloy. However, α also reflects the fracture ductility of the alloy, and it can be taken as a new fracture-ductility parameter. When the limit of α changes, the relationship among yield stress-strain, stress coefficient, and strain-hardening exponent changes, too. So with the aid of α, the applicability of the relationship among the four material constants for different metals can be determined.

Going a step further, the alloy strain-hardening behavior[9,10] is generally described by the value of $\sigma_b/\sigma_{0.2}$. When $(\sigma_b/\sigma_{0.2}) > 1.4$, the alloy behaves in a cyclic-hardening manner. When $(\sigma_b/\sigma_{0.2}) < 1.2$, the alloy behaves in a cyclic-softening manner. When $1.2 < (\sigma_b/\sigma_{0.2}) < 1.4$, the alloy behaves in an indeterminate manner. Figure 1 and 2 show the σ-ε cyclic curves and the σ-ε tensile curves[5] corresponding to alloys Lc4cs and Lc9cgs3. The stress-strain curves of the other alloys listed in Tables 1 and 2 can be found in Ref 5-7. The hardening behavior of the alloys is noted as *H/S,* and these values are listed in the two tables. It can be seen from the figures and the tables that for Lc4cs and for Lc9cgs3, the values of $(\sigma_b/\sigma_{0.2})$ are 1.08, but that they behave in a cyclic-hardening manner.

If the value of α is related to the hardening behavior of the alloy, it can be seen that when $\alpha > 20\%$, the alloy behaves in a cyclic-softening manner, but when $\alpha < 20\%$, the alloy behaves in a cyclic-hardening manner. Therefore, calculating the fracture-ductility parameter α may provide a better way for describing the hardening behavior of metals and alloys, rather than using the ratio $(\sigma_b/\sigma_{0.2})$.

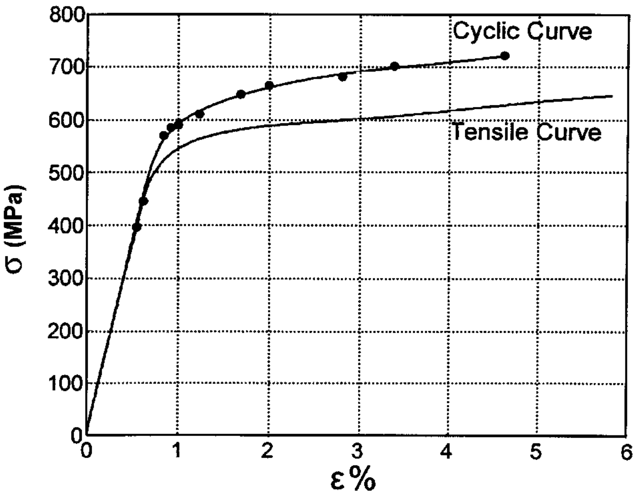

Fig. 1 σ-ε curves for Lc4cs[5]

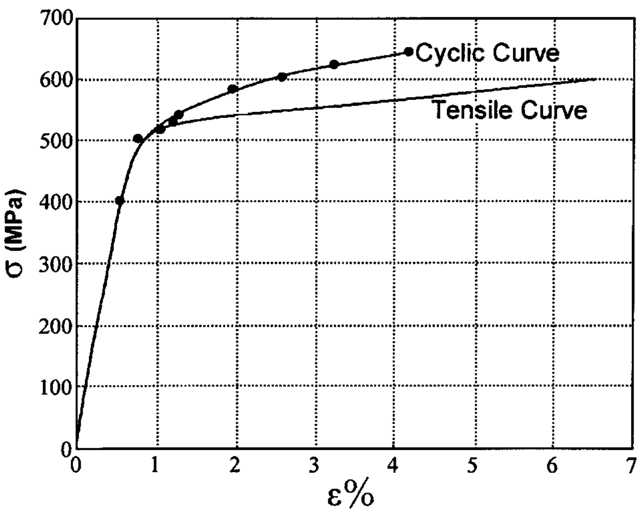

Fig. 2 σ-ε curves for Lc9cgs3[5]

4. Conclusions

Studying the mechanical behavior of selected alloys in the present paper, the following conclusions can be made.

A new fracture ductility parameter α has been introduced:

$$\alpha = \psi\, \varepsilon_f = -\psi \ln(1 - \psi)$$

On the one hand, the new fracture-ductility parameter can be used to describe the applicability of the equations derived in this paper. On the other hand, it can be used to express the hardening behavior of alloys.

The expressions that reflect the relationship among yield stress-strain, strength coefficient, and strain-hardening exponent (when $k < \sigma_f$, or 5% < α <10%, or α > 20%) are:

$$\sigma_{0.2}^{3/2} = \sigma_b^{1/2}\, k(0.002)^n$$

However, when $k > \sigma_f$, or α < 5%, or 10% < α < 20%, the expression becomes:

$$\sigma_{0.2}^{5/3} = \sigma_b^{2/3}\, k(0.002)^n$$

When α > 20%, the alloy behaves in a cyclic-softening manner; but when α < 20%, the alloy behaves in a cyclic-hardening manner.

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